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Financial Time Series: Nonstationarity and Volatility

A financial time series exhibits two central properties: nonstationarity and time-varying volatility. In a broad sense, nonstationarity means that a variable has no clear tendency to return to a constant value or linear trend.



The figure above shows a daily series: the value of the British Pound expressed in US Dollars. This exchange rate series does not seem to be stationary, i.e., return to a fixed value or fluctuate around a linear trend... The second central property of a financial time series is that its volatility varies over time. Volatility is defined as the first difference of a series.

r\_t = log(x\_t) - log(x\_{t-1})

An important objective of research in time series analysis is to test hypotheses and estimate relationship amongst such variables. However, inferences drawn from a stationary analysis would not be valid if the series being examined are indeed realizations of nonstationary processes.

What is a Time Series?

A time series is a sequence of measurements through time. Mathematically, a time series may be represented as X = {x\_t | t = 1, ..., N}

where t is the time index and N is the total number of observations. A financial time series is a sequence of numbers related to the value of a financial instrument, e.g. currency, shares and so on; a meteorological time series is a sequence of numbers related to weather parameters, e.g. temperature, pressure, humidity etc.

Introduction

Analysing and Reporting Financial Markets: How Experts Summarize a Time Series?



The figure to the left shows an excerpt from FINANCIAL TIMES (FT.COM). The time series shown is the composite German index, the DAX, while surrounding it are the various numeric data associated with the index.

Experts study the behaviour of such time series by analysing the various numeric data associated with it. For example, the time series shown here may have some chief features like, cycles, turning points, trends and volatility patterns which could have bearing on its future behaviour and value.

Research Objective

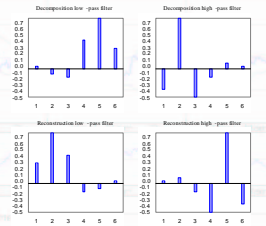
We believe that nonstationarity and time-varying volatility are the two most important inherent characteristics of a financial time series. Therefore, any attempt to analyse and summarise such time series data needs to take into account these two central properties.

Our research objective is to automatically summarise nonstationary and volatile financial time series data with respect to 'chief features' like cycles, trends, turning points and variance change.

The Approach

Separating Slow and Rapidly Moving Time Series: Wavelet Filtering

Recently it has been claimed that wavelet filtering provides insight into the dynamics of financial time series beyond that of current classical statistical methodology. The figure below shows the Daubechies six-coefficient wavelet filters.



A number of concepts, for example nonstationarity, multiresolution, and approximate decorrelation have emerged from wavelet filters [2]. Wavelet analysis decomposes a time series into several sub-series (approximation and details), which may be associated with particular time scales. The interpretation of features in complex financial time series is made easy by first applying the wavelet transform and subsequently interpreting each individual sub-series.

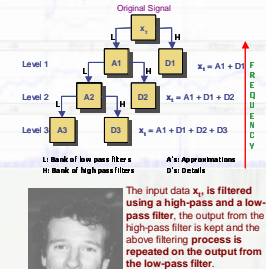
Daubechies, L., 'Ten Lectures on Wavelets', SIAM, Philadelphia, 1992.

Discrete Wavelet Transform (DWT)

The discrete wavelet transform (DWT) can be seen as a repeated convolution process. The discrete convolution process can be expressed by the following formula:

W \* x\_t = sum\_{k=-infinity}^{infinity} W\_k x\_{t-k}

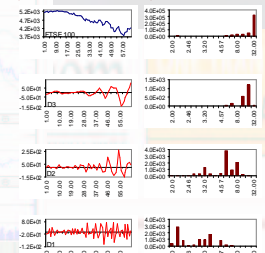
Here x\_t is the original signal while w is the low or high-pass filter corresponding to the prototype wavelet (for example, Daubechies six-coefficient wavelet filters). The DWT is implemented using Mallat's pyramidal algorithm. This approach is shown below:



Mallat, S., 'A Theory for Multiresolution Signal Decomposition: The Wavelet Representation', IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 11, p. 674-683, July 1989.

DWT and the Fourier Power Spectrum

The figure below shows a level-3 'db3' DWT multiresolution analysis (MRA) of the daily value of the FTSE 100 index for a period of 63 days. The price series is plotted in the top row (blue) while below it - from top to bottom - are the wavelet details D\_3, D\_2 and D\_1 (red). In the right panel are the corresponding Fourier Power Spectra of the series in the left panel.



According to Fourier analysis, any finite time series x\_t can be represented as a combination of several sinusoids having different amplitudes and frequencies.

The equation given below is known as Parseval's relation where the squared magnitude of the Fourier transform |X\_k|^2 is known as the Fourier power spectrum of the original signal x\_t.

sum\_{k=0}^{N-1} |X\_k|^2 = 1/N sum\_{k=0}^{N-1} |x\_k|^2

Hence a Fourier power spectrum is the distribution of a signal with respect to its frequency and its peak value would indicate the most dominant cycle of the analyzed signal.

Consider again the figure to the left. In wavelet details D\_3 there is a noticeable fluctuation from the 40th day onwards and its corresponding Fourier power spectrum indicates a peak at 16. This implies that a 16-day cycle has started on the 40th day in the FTSE 100 series and this might persist.

Statistical NCSS

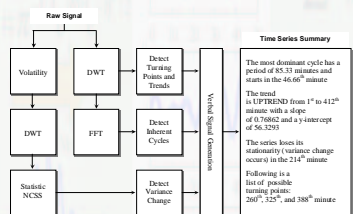
In the so-called long memory processes, the correlations between variables do not decay at a sufficiently fast rate. The normalised cumulative sum of squares (NCSS) index can be used to test for homogeneity of variances, on a scale-by-scale basis, for such long memory processes [4]. This provides a statistically sound technique of testing for nonstationary features without knowing the exact nature of the correlation structure in a given time series. It is defined as

P\_k = sum\_{l=L-k}^{L-1} sum\_{j=l}^{L-1} W\_{j,t}^2 / sum\_{j=l}^{L-1} W\_{j,t}^2, k=L-1, ..., N-2

where N is the total number of samples, L is the length of the filter used for the DWT analysis and W\_j is the level-j DWT of the volatility series v\_t.

The Outcome: A Prototype for Summarizing Volatile Financial Time Series

Using the mathematical theories discussed, we have developed a prototype system that takes into account nonstationarity, volatility and long-range dependencies, to automatically extract chief features from a time series and generates a verbal description of each of the extracted features [1]. System architecture is shown in figure below.



Our system uses a combination of the discrete wavelet transform (DWT) and the fast Fourier transform (FFT) to extract all the features from a time series. The functionality of the system can be summed up in the following steps:

- 1. The FFT power spectra of the DWT sub-series help us to detect and locate cycles in the original signal.
2. The extrema of the DWT sub-series detect major edges in the signal and hence correspond to most of the turning points [3].
3. The recursive DWT filtering process removes long and short-term fluctuations from the signal in each recursion to give a linear trend in the end.
4. The variance change location is detected using the DWT on the volatility time series and making use of the so-called normalised cumulative sum of squares (NCSS) index.
5. Once numerical information (outlying the trend, cycles, turning points and variance change) is extracted from raw data, it is jelled with predefined strings to generate a meaningful summary of the input time series x\_t.

Initial experiments with our prototype have been encouraging and we plan to carry this research forward to improve the efficacy of our methods and algorithms.

IBM Prices: System Output for Cycles, Trend, Turning Points and Variance Change

We present results of a study performed on the daily IBM prices from May 17, 1961 to November 2, 1962. This is the classic data studied in Box and Jenkins [2]. Top left corner of the figure below shows the original series. The other three portions of the figure show the various outputs from our system.

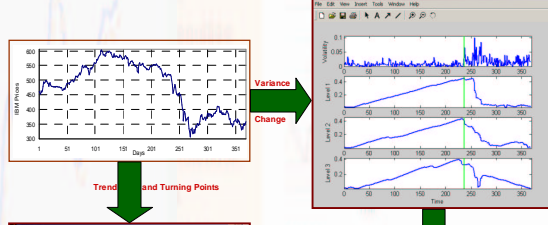


Table with 2 columns: Feature and IBM Series (May 17, 1961-November 2, 1962). Rows include Trend (Downtrend), Dominant Cycle (85.3 days), Key Turning Points (113th, 270th, and 321st day), and Variance Change (Occurs on the 23rd day).

The outputs shown here visualize the trend, turning points and variance change. The turning points are marked red on the time series (bottom left portion). The trend is a downtrend and is shown with a falling green line (bottom left portion). For variance change, the volatility of the original series is shown followed by three levels of the NCSS index (top right portion). The peak value of the NCSS index detects where exactly a variance change occurs or where the series loses its stationarity. All these features are summarized by our system by generating verbal signals for each feature (bottom right portion). We have not shown the visualization of cycles for this series, however, the verbal signal generated describes it as 85.3 days, starting on the 46th day.

References

- [1] Ahmad, S., Oliveira, P. and Ahmad, K., 'Summarization of Multimedial Information' (to appear in the proceedings of IJREC-2004, Lisbon, Portugal)
[2] Box, G. E. P., Jenkins, G. M., 'An Introduction to the Theory of Time Series Analysis', Academic Press, 2002.
[3] Marr, D., 'Vision: A Computational Investigation into the Human Representation and Processing of Visual Information', W. H. Freeman, San Francisco, 1982.
[4] Whitcher, B., Percival, D. B. and Guttorp, P., 'Multiscale Detection and Location of Multiple Variance Changes in the Presence of Long Memory', Journal of Statistical Computation and Simulation, Vol. 68, pp. 65-89, 2000.

Acknowledgments

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